PHYSICAL REVIEW D **80,** 025006 (2009)

Possible treatment of the ghost states in the Lee-Wick standard model

Abouzeid M. Shalaby[1,2,*]

[1]*Physics Department, Faculty of Science, Mansoura University, Egypt*
[2]*Physics Department, Faculty of Science, Qassim University, Saudi Arabia*
(Received 20 January 2009; published 13 July 2009)

In this work, we employ the techniques used to cure the indefinite norm problem in pseudo-Hermitian Hamiltonians to show that the ghost states in a higher derivative scalar field theory are not real ghosts. For the model under investigation, an imaginary auxiliary field is introduced to have an equivalent non-Hermitian two-field scalar theory. We were able to calculate exactly the positive definite metric operator η for the quantum mechanical as well as the quantum field versions of the theory. While the equivalent Hamiltonian is non-Hermitian in a Hilbert space characterized by the Dirac sense inner product, it is, however, a Hermitian in a Hilbert space endowed with the inner product $\langle n|\eta|m\rangle$. The main feature of the latter Hilbert space is that the propagator has the correct sign (no Lee-Wick fields). Moreover, the calculated metric operator diagonalizes the Hamiltonian in the two fields (no mixing). We found that the Hermiticity of the calculated metric operator to lead to the constrain $M > 2m$ for the two Higgs masses, in agreement with other calculations in the literature. Besides, our mass formulas coincide with those obtained in other works (obtained by a very different regime but with the existence of ghost states), which means that our positive normed Hamiltonian form preserves the mass spectra.

DOI: 10.1103/PhysRevD.80.025006 PACS numbers: 12.90.+b, 11.30.Er, 12.60.Cn

The origin of the mass of the building blocks in our universe represents one of the greatest puzzles in the theory of particle interactions. The puzzle comes from the existence of a conceptual problem in the conventional standard model regarding the flow of the dimensionful parameters at high energy scales. At these scales, the mass as well as all the dimensionful parameters of the Higgs particle flow up to unacceptable very large values. This unacceptable flow of the parameters is known in the literature as the gauge hierarchy problem [1]. On the other hand, it is well known that models with supersymmetry (SUSY) do not suffer from the hierarchy problem. In these models, natural cancellations occur between cutoff dependent terms, resulting from fermion and boson loops. These cancellations, in turn, protect the problematic parameters against perturbations up to very high energy scales [2]. However, SUSY introduces an upper limit to the Higgs mass by 130 GeV and some of its mass spectra are of 1 TeV. These features expose SUSY to the direct fire of the LHC tests. Accordingly, we may need an alternative scenario for the description of particle interactions, which avoid the above mentioned problems and limitations.

Very recently, with the guidance of a previous work of Lee and Wick (Lee-Wick or LW) [3,4], a non-SUSY extension of the standard model has been introduced and investigated [5–8]. While the LW QED is a finite theory, the non-Abelian LW gauge theory is not a finite one. However, the Lee-Wick standard model, as a non-Abelian LW gauge theory, is renormalizable and it does solve the hierarchy problem. In fact, in the LW extension of the standard model, every field of the conventional standard model has a higher derivative kinetic term. Also, it has been shown that the higher derivative theory can be converted into an equivalent ordinary one with more fields. Out of these fields, the LW one has a propagator with a wrong sign (exotic). Accordingly, the natural cancellation of cutoff dependent terms can occur between terms that come from loops involving normal fields and those which come from exotic fields.

The main idea of any Lee-Wick model is to consider two kinds of fields with kinetic terms of opposite signs. The field of negative kinetic term is known as the LW field. In fact, one can show that a one-field theory with a higher derivative term can have an equivalent structure to that of a Lee-Wick model [5,6]. The existence of the Lee-Wick fields has benefits such as QED finiteness and hierarchy absence. Nevertheless, the presence of the exotic fields (LW fields) resembles a great puzzle in the Lee-Wick theory that introduces a problem concerning the probabilistic interpretation in the theory. Accordingly, in taking into account the successes in the LW theories like QED finiteness, for example, it becomes an important challenge to solve the indefinite norm problem existing in the Lee-Wick theories.

In a very different kind of study, Bender and Mannheim have shown that a quantum mechanical theory with higher derivatives, which apparently suffers from the indefinite norm problem, can be converted into an equivalent one without any ghost states [9]. In showing that, they stressed a higher derivative Pais-Uhlenbeck model. In fact, the indefinite norm problem is common in $\mathcal{PT}$-symmetric theories. However, there exist well-known algorithms to cure ghost states existing in pseudo-Hermitian theories [10–14]. It is noteworthy to know that the $\mathcal{PT}$ symmetry

*amshalab@mans.edu.eg

(exact) is sufficient, but not necessary, for the existence of a real spectrum in a non-Hermitian theory. In fact, Mostafazadeh has shown that the reality of the spectrum of a Hamiltonian model is not limited to either Hermiticity or $\mathcal{PT}$ symmetricity [11,12]. Instead, he proved that if a Hamiltonian model H has the property $\eta H \eta^{-1} = H^{\dagger}$, i.e. H is pseudo-Hermitian, then the spectrum of H is real. Here η is a Hermitian, linear, invertible, and positive definite operator. This formulation of the problem can be applied to Hermitian theories ($\eta = I$), $\mathcal{PT}$-symmetric theories, and any non-Hermitian Hamiltonian for which a positive definite metric operator exists. With this in mind, in this work, we show that the techniques applied to pseudo-Hermitian theories can be successfully applied, in principle, to the different sectors of the recently introduced LW standard model. In fact, as a starting point, we shall stress a type of scalar field theory that is very similar to the one employed in the Lee-Wick standard model. Using that model, we show that the theory is free from ghost states. Since the used model is a prototype example of the sectors in the Lee-Wick standard model, we expect that the algorithm can be safely and successfully extended to the whole theory.

To start, we consider the prototype scalar field Lagrangian introduced in the Lee-Wick standard model of the form [5]

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}(\partial^2\phi)^2 - \frac{1}{2}m^2\phi^2. \qquad (1)$$

Following the work in Ref. [5], one can introduce an auxiliary field ϕ_2 to get rid of the higher derivative in the theory such that

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2 - \phi_2\partial^2\phi + \tfrac{1}{2}M^2\phi_2^2. \qquad (2)$$

From the equation of motion of ϕ_2, we get

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_2)} = 0, \qquad \frac{\partial\mathcal{L}}{\partial\phi_2} = \phi_2 M^2 - \partial^2\phi.$$

Then, the auxiliary field ϕ_2 is given by the relation

$$\phi_2 = \frac{1}{M^2}\partial^2\phi.$$

Let us define the field ϕ_1 through the relation

$$\phi = \phi_1 - \phi_2;$$

then

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\partial_\mu(\phi_1 - \phi_2)\partial^\mu(\phi_1 - \phi_2) - \tfrac{1}{2}m^2(\phi_1 - \phi_2)^2 \\ &\quad - \phi_2\partial^2(\phi_1 - \phi_2) + \tfrac{1}{2}M^2\phi_2^2, \\ &= \tfrac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \tfrac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \partial_\mu\phi_2\partial^\mu\phi_1 - \phi_2\partial^2\phi_1 \\ &\quad + \phi_2\partial^2\phi_2 - \tfrac{1}{2}m^2(\phi_1 - \phi_2)^2 + \tfrac{1}{2}M^2\phi_2^2, \\ &= \tfrac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \tfrac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \tfrac{1}{2}m^2\phi_1^2 \\ &\quad + \tfrac{1}{2}(M^2 - m^2)\phi_2^2 + m^2\phi_2\phi_1. \end{aligned} \qquad (3)$$

The Hamiltonian corresponding to the Lagrangian in Eq. (3) can be obtained as

$$\begin{aligned} H &= \frac{\pi_1^2}{2} + \frac{1}{2}(\nabla\phi_1)^2 + \frac{1}{2}m^2\phi_1^2 - \frac{\pi_2^2}{2} - \frac{1}{2}(\nabla\phi_2)^2 \\ &\quad - \frac{1}{2}(M^2 - m^2)\phi_2^2 - m^2\phi_1\phi_2. \end{aligned} \qquad (4)$$

Now, let us apply the canonical transformation $\phi_2 \to i\phi_2$, $\pi_2 \to -i\pi_2$, which preserves the commutation relations [9],

$$[\phi_2(x), \pi_2(y)] = [i\phi_2(x), -i\pi_2(y)] = i\delta^3(x - y). \qquad (5)$$

Then the transformed Hamiltonian will take the form,

$$\begin{aligned} H &= \frac{\pi_1^2}{2} + \frac{1}{2}(\nabla\phi_1)^2 + \frac{1}{2}m^2\phi_1^2 + \frac{\pi_2^2}{2} + \frac{1}{2}(\nabla\phi_2)^2 \\ &\quad + \frac{1}{2}(M^2 - m^2)\phi_2^2 - im^2\phi_1\phi_2. \end{aligned} \qquad (6)$$

In other words, the negative norm manifested in the work of Ref. [5] by a negative kinetic term of the LW field (ϕ_2) is manifested here by the non-Hermiticity of the Hamiltonian represented by the form in Eq. (6). By assuming that ϕ_2 is pseudo scalar, the Hamiltonian obtained in Eq. (6) is $\mathcal{PT}$ symmetric, too. Indeed, the non-Hermitian and $\mathcal{PT}$-symmetric theories are suffering from the existence of ghost states. However, there exists known algorithms to recover such problems [9–11]. In fact, though the Hamiltonian in Eq. (6) is non-Hermitian in the Dirac sense, it is not only Hermitian in a Hilbert space endowed by the inner product $\langle n|\eta|m\rangle$ [11,12], but it also has kinetic terms in the correct form (no LW fields). To build up the Hilbert space with the inner product $\langle n|\eta|m\rangle$, we need to obtain the metric operator η for the theory under investigation. In the following, we show how to obtain η in a closed form.

To start the algorithm of curing the ghost states problem in the theory, we stress the theory in $0 + 1$ dimensions (quantum mechanics). Since the Hamiltonian in Eq. (6) is pseudo-Hermitian, one can seek a positive definite metric operator of the form

$$\eta = \exp(2(\omega_1\pi_1\phi_2 + \omega_2\pi_2\phi_1)),$$

where ω_1 and ω_2 are two real parameters to be obtained later in terms of the mass parameters m and M. Note that η is Hermitian and has the property [11,12]

$$\eta H \eta^{-1} = H^{\dagger}. \qquad (7)$$

Also, $\rho = \sqrt{\eta}$ has the property

$$\rho H \rho^{-1} = h, \qquad (8)$$

where h is a Hermitian (in the Dirac sense) as well as the positive normed Hamiltonian equivalent to H.

To determine the parameters ω_1 and ω_2, we consider the transformations of the different fields in the Hamiltonian under the effect of ρ as follows:

$$\rho\phi_1\rho^{-1} = \phi_1 - i\omega_1\phi_2, \qquad \rho\pi_1\rho^{-1} = \pi_1 + i\omega_2\pi_2,$$
$$\rho\phi_2\rho^{-1} = \phi_2 - i\omega_2\phi_1, \qquad \rho\pi_2\rho^{-1} = \pi_2 + i\omega_1\pi_1.$$

Accordingly,

$$h = \frac{(\pi_1 + i\omega_2\pi_2)^2}{2} + \frac{1}{2}m^2(\phi_1 - i\omega_1\phi_2)^2 + \frac{(\pi_2 + i\omega_1\pi_1)^2}{2} + \frac{1}{2}(M^2 - m^2)(\phi_2 - i\omega_2\phi_1)^2 - im^2(\phi_1 - i\omega_1\phi_2)(\phi_2 - i\omega_2\phi_1),$$

or

$$\begin{aligned} h = {} & \tfrac{1}{2}\pi_1^2 + i\pi_1\omega_2\pi_2 - \tfrac{1}{2}\omega_2^2\pi_2^2 + \tfrac{1}{2}m^2\phi_1^2 - \tfrac{1}{2}m^2\omega_1^2\phi_2^2 \\ & + \tfrac{1}{2}\pi_2^2 + i\pi_2\omega_1\pi_1 - \tfrac{1}{2}\omega_1^2\pi_1^2\tfrac{1}{2}(M^2 - m^2) \\ & \times (\phi_2^2 - \omega_2^2\phi_1^2) - m^2\omega_2\phi_1^2 - m^2\omega_1\phi_2^2 \\ & - im^2\phi_1\phi_2 + im^2\omega_1\phi_2\omega_2\phi_1 - i\phi_2\omega_2\phi_1 M^2 \\ & + i\phi_2\omega_2\phi_1 m^2 - im^2\phi_1\omega_1\phi_2. \end{aligned} \tag{9}$$

For h to be Hermitian, one has to put constraints,

$$\begin{aligned} i\omega_2 + i\omega_1 = 0, \\ (-m^2 + m^2\omega_1\omega_2 - \omega_2 M^2 + \omega_2 m^2 - m^2\omega_1) = 0, \end{aligned} \tag{10}$$

on the introduced parameters ω_1 and ω_2. Equivalently, we have the relations

$$\omega_1 = -\omega_2, \qquad -m^2 - m^2\omega_1^2 + \omega_1 M^2 - 2m^2\omega_1 = 0. \tag{11}$$

In terms of the mass parameters, ω_1 can be obtained as

$$\omega_1 = \frac{1}{2m^2}(M^2 - 2m^2 \pm \sqrt{M^4 - 4M^2m^2}). \tag{12}$$

Also, due to the reality of ω_1, the two Higgs masses are related by

$$M^2 \geq 4m^2,$$

which agrees with the results in Ref. [5].

Now, in view of the above constraints on the parameters ω_1 and ω_2, the Hermitian Hamiltonian h has the form

$$\begin{aligned} h = {} & \tfrac{1}{2}\pi_1^2(1 - \omega_1^2) + \tfrac{1}{2}m^2\phi_1^2 + \tfrac{1}{2}(1 - \omega_1^2)\pi_2^2 - \tfrac{1}{2}m^2\omega_1^2\phi_2^2 \\ & + \tfrac{1}{2}(M^2 - m^2)(\phi_2^2 - \omega_1^2\phi_1^2) + m^2\omega_1\phi_1^2 - m^2\omega_1\phi_2^2, \\ = {} & \tfrac{1}{2}\pi_1^2(1 - \omega_1^2) + \tfrac{1}{2}m^2\phi_1^2 + \tfrac{1}{2}(1 - \omega_1^2)\pi_2^2 \\ & + \Big(\tfrac{1}{2}m^2\omega_1^2 + m^2\omega_1 - \tfrac{1}{2}M^2\omega_1^2\Big)\phi_1^2 \\ & + \Big(-\tfrac{1}{2}m^2\omega_1^2 + \tfrac{1}{2}M^2 - m^2\omega_1 - \tfrac{1}{2}m^2\Big)\phi_2^2. \end{aligned} \tag{13}$$

To make sure that the negative norm problem has been lifted, we plotted the propagator-sign governing factors of the form $\mu_0^2 = (1 - \omega_1^2)$, $\mu_1^2 = (\frac{1}{2}m^2\omega_1^2 + m^2\omega_1 - \frac{1}{2}M^2\omega_1^2)$, and $\mu_2^2 = (-\frac{1}{2}m^2\omega_1^2 + \frac{1}{2}M^2 - m^2\omega_1 - \frac{1}{2}m^2)$ as a function of M for $m = 1$, shown in Figs. 1–3, respectively. In these plots, we have taken the root $\omega_1 = \frac{1}{2m^2} \times (M^2 - 2m^2 - \sqrt{M^4 - 4M^2m^2})$, while the other root represents a theory of indefinite norm. One can realize that all these factors are positive for the available range of M, which assures the remedy of the wrong sign in the propagator of the LW field.

To show that the transformations carried out preserve the mass spectra of the original theory, we compare the mass formula to those obtained in Ref. [6]. To do that, we apply the canonical transformations of the form

$$\psi_1 = \frac{1}{\sqrt{1 - \omega_1^2}}\phi_1, \qquad \Pi_1 = \sqrt{(1 - \omega_1^2)}\pi_1, \tag{14}$$

$$\psi_2 = \frac{1}{\sqrt{1 - \omega_1^2}}\phi_2, \qquad \Pi_2 = \sqrt{(1 - \omega_1^2)}\pi_2, \tag{15}$$

to Eq. (13) and to note that the second relation in Eq. (11) can be written as

$$m^2(\omega_1 + 1)^2 = \omega_1 M^2. \tag{16}$$

Also, we can write

$$\omega_1 + 1 = \frac{M^2}{2m^2}\left(1 - \sqrt{1 - \frac{4m^2}{M^2}}\right), \tag{17}$$

and thus we get the mass formula for the field ψ_1 as

$$m_{\psi_1}^2 = (\omega_1 + 1)m^2 = \frac{M^2}{2}\left(1 - \sqrt{1 - \frac{4m^2}{M^2}}\right), \tag{18}$$

which is the same formula obtained in Eq. (8) of Ref. [6]. These results show that the obtained Hamiltonian in our

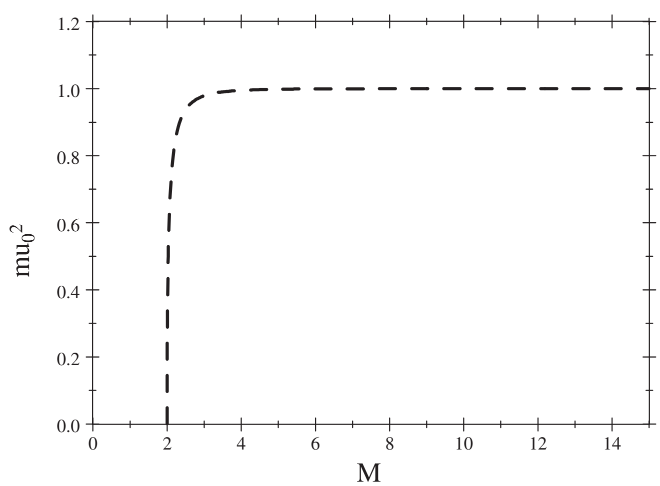

FIG. 1. The factor $\mu_0^2 = (1 - \omega^2)$ plotted against the mass parameter M for $m = 1$. One can realize that the factor is positive for the available range of M.

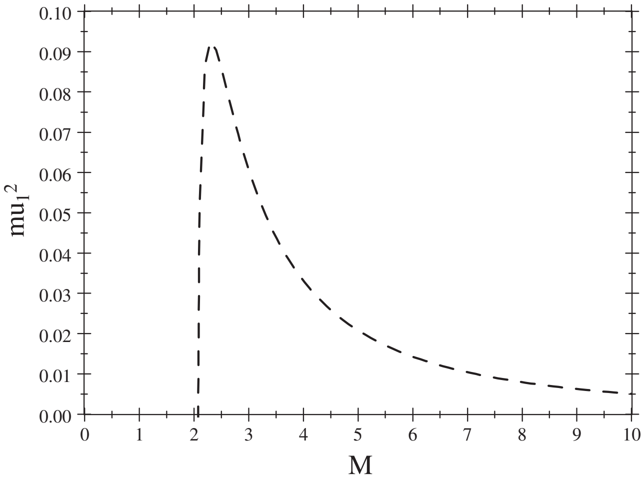

FIG. 2. A contribution to the mass parameter squared of the field ϕ_1 of the form $\mu_1^2 = (\frac{1}{2}m^2\omega_1^2 + m^2\omega_1 - \frac{1}{2}M^2\omega_1^2)$ in the Hermitian Hamiltonian h, plotted against the mass parameter M for $m = 1$. Since the other contribution is m^2 and from the plot μ_1^2 is always positive, then the mass squared as a whole is positive.

work has no ghosts and preserves the spectrum of the original theory.

In higher dimensions (quantum field theory), one needs to deal with operator densities, and thus the metric operator will take the from

$$\eta = \int d^3z \exp(2(\omega_1\pi_1(z)\phi_2(z) + \omega_2\pi_2(z)\phi_1(z))).$$

Accordingly, we have the relations

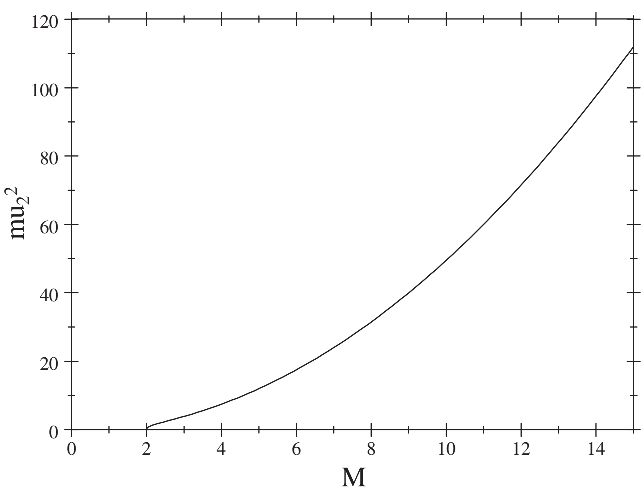

FIG. 3. The mass parameter squared of the field ϕ_2 given by $\mu_2^2 = -\frac{1}{2}m^2\omega_1^2 + \frac{1}{2}M^2 - m^2\omega_1 - \frac{1}{2}m^2$ in the Hermitian Hamiltonian h, plotted against the mass parameter M for $m = 1$, which is again a positive quantity.

$$\rho\phi_1(x)\rho^{-1} = \phi_1(x) - i\omega_1\int d^3z\phi_2(z)\delta^3(x-z),$$
$$\rho\pi_1\rho^{-1} = \pi_1 + i\omega_2\int d^3z\pi_2(z)\delta^3(x-z),$$
$$\rho\phi_2^{-1}\rho^{-1} = \phi_2 - i\omega_2\int d^3z\phi_1(z)\delta^3(x-z),$$
$$\rho\pi_2^{-1}\rho^{-1} = \pi_2 + i\omega_1\int d^3z\pi_1(z)\delta^3(x-z).$$

And thus

$$\rho\phi_1^{-1}\rho^{-1} = \phi_1 - i\omega_1\phi_2,$$
$$\rho\pi_1^{-1}\rho^{-1} = \pi_1 + i\omega_2\pi_2,$$
$$\rho\phi_2^{-1}\rho^{-1} = \phi_2 - i\omega_2\phi_1,$$
$$\rho\pi_2^{-1}\rho^{-1} = \pi_2 + i\omega_1\pi_1.$$

Also, note that

$$\begin{aligned}\rho\frac{1}{2}(\nabla\phi_1(x))^2\rho^{-1} &= \frac{1}{2}(\nabla\phi_1(x))^2 - i\omega_1\nabla_x\phi_1(x)\nabla_x\phi_2(x) - \frac{\omega_1^2}{2}(\nabla_x\phi_2(x))^2,\\ \rho\frac{1}{2}(\nabla\phi_2(x))^2\rho^{-1} &= \frac{1}{2}(\nabla\phi_2(x))^2 - i\omega_2\nabla_x\phi_1(x)\nabla_x\phi_2(x) - \frac{\omega_2^2}{2}(\nabla_x\phi_1(x))^2.\end{aligned} \tag{19}$$

Again, with the choice $\omega_1 = -\omega_2 = \omega$, one gets

$$\begin{aligned}&\rho(\tfrac{1}{2}(\nabla\phi_1(x))^2 + \tfrac{1}{2}(\nabla\phi_2(x))^2)\rho^{-1}\\ &\quad = \tfrac{1}{2}(1-\omega^2)(\nabla\phi_1(x))^2 + (\nabla\phi_2(x))^2,\end{aligned} \tag{20}$$

and the quantum field Hermitian Hamiltonian takes the form

$$\begin{aligned}h = {}&\tfrac{1}{2}\pi_1^2(1-\omega^2) + \tfrac{1}{2}(1-\omega^2)(\nabla\phi_1(x))^2 + \tfrac{1}{2}m^2\phi_1^2\\ &+ \tfrac{1}{2}(1-\omega^2)\pi_2^2 + \tfrac{1}{2}(1-\omega^2)(\nabla\phi_2(x))^2 + \tfrac{1}{2}M^2\phi_2^2\\ &+ (\tfrac{1}{2}m^2\omega^2 + m^2\omega - \tfrac{1}{2}M^2\omega_1^2)\phi_1^2\\ &+ (-\tfrac{1}{2}m^2\omega_1^2 - m^2\omega_1 - \tfrac{1}{2}m^2)\phi_2^2.\end{aligned} \tag{21}$$

One can easily realize that the governing factors are all positive for the available range of the mass parameter M relative to the mass parameter m. Accordingly, the problem of the wrong sign propagator has been recovered. Another benefit of the transformation mapping $H \rightarrow h$ is that there exists no mixing terms in h (h is diagonal in the fields ϕ_1 and ϕ_2).

To make sure that the Hermitian equivalent Hamiltonian in Eq. (21) still bears the feature of quadratic divergence cancellation, we rewrite it in the form

$$h = h_1 + h_2,$$
$$h_1 = \tfrac{1}{2}(\pi_1^2 + (\nabla\phi_1)^2) + \tfrac{1}{2}m^2(1+\omega_1)^2\phi_1^2 + \tfrac{1}{2}(-\omega_1^2)(\pi_2^2 + (\nabla\phi_2)^2) + \tfrac{1}{2}(-m^2(1+\omega_1)^2)\phi_2^2,$$
$$h_2 = \tfrac{1}{2}(\pi_2^2 + (\nabla\phi_2)^2) + \tfrac{1}{2}M^2\phi_2^2 + \tfrac{1}{2}(-\omega_1^2)(\pi_1^2 + (\nabla\phi_1)^2) + \tfrac{1}{2}(-M^2\omega_1^2)\phi_1^2,$$

which shows that although h is a Hermitian and positive normed, it can be decomposed into two terms each of which has the form of the normal and Lee-Wick fields.

In conclusion, we considered a higher derivative scalar field theory of the form used in the Lee-Wick standard model. For the theory considered and via a simple canonical transformation, we obtained a non-Hermitian but $\mathcal{PT}$-symmetric two-field equivalent Hamiltonian. Using the tools applied to cure the indefinite norm problem in pseudo-Hermitian Hamiltonians, we were able to obtain the positive definite metric operator for both the quantum mechanical and quantum field versions of the theory in a closed form. We showed that the obtained equivalent Hermitian Hamiltonian has propagators of the correct sign, which means that the ghost problem has been cured. Moreover, the Hermitian Hamiltonian is diagonal in the fields. To test the validity of our results, we showed that the Hermiticity of the calculated metric operator to lead to the constrain $M > 2m$ for the two Higgs masses, in agreement with other calculations in the literature. Moreover, our mass formulas coincide with those obtained in other works obtained in Ref. [6]. Note that we discarded the potential term since it is used to break the symmetry and has no effect on the negative norm of the auxiliary field, and thus one can instead add it to the equivalent Hermitian Hamiltonian. Indeed, one can deal with the whole theory including the potential term (non-Hermitian). In this case, the metric operator can be calculated perturbatively [15]. However, we assert that the work presented here is fully new and it is the first time the exact metric operator for a realistic quantum field theory was obtained.

For the other sectors in the Lee-Wick standard model, there exits a higher derivative term which could be absorbed with the appearance of more fields like the scalar sector we stressed in this work. One may be able, in principle, to go the same way we followed in this work to cure the ghost states in the full theory. This kind of calculation will take a substantial amount of time. Naturally, the metric operator calculation for more realistic higher derivative theories will become a topic of our future work with an ultimate aim to obtain a Lee-Wick standard model with no ghosts.

A note to be mentioned is that the mass of the auxiliary field is greater than the normal Higgs mass, which means that it is out of any experimental tests carried out.

I am very grateful to Dr. S. A. Elwakil for his support and would like to thank Dr. B. A. Shehadeh for his help in revising the manuscript.

[1] Abdelhak Djouadi, Phys. Rept. **457**, 1 (2008).
[2] J. Wess and B. Zumino, Nucl. Phys. **B70**, 39(1974).
[3] T. D. Lee and G. C. Wick, Nucl. Phys. **B9**, 209 (1969).
[4] T. D. Lee and G. C. Wick, Phys. Rev. D **2**, 1033 (1970).
[5] Benjamín Grinstein, Donal O'Connell, and Mark B. Wise, Phys. Rev. D **77**, 025012 (2008).
[6] C. D. Carone and R. F. Lebed, Phys. Lett. B **668**, 221 (2008).
[7] Thomas G. Rizzo, J. High Energy Phys. 06 (2007) 070.
[8] José Ramón Espinosa, Benjamín Grinstein, Donal O'Connell, and Mark B. Wise, Phys. Rev. D **77**, 085002 (2008).
[9] Carl M. Bender and Philip D. Mannheim, Phys. Rev. Lett. **100**, 110402 (2008).
[10] Carl M. Bender, Sebastian F. Brandt, Jun-Hua Chen, and Qing-hai Wang, Phys. Rev. D **71**, 025014 (2005).
[11] A. Mostafazadeh, J. Math. Phys. (N.Y.) **43**, 3944 (2002).
[12] A. Mostafazadeh, J. Math. Phys. (N.Y.) **43**, 205 (2002).
[13] H. F. Jones and R. J. Rivers, Phys. Rev. D **75**, 025023 (2007).
[14] Carl Bender and Stefan Boettcher, Phys. Rev. Lett. **80**, 5243 (1998).
[15] Abouzeid Shalaby, Phys. Rev. D **79**, 107702 (2009).